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Philosophy 666: Rational Choice Theory
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# Test on Utility Theory—Answer Key

***Instructions:***

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period—no credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

***Questions:***

1. What does the transitivity condition require? (You do not need to state all four clauses; you can just state the first one.) What is the money-pump argument? How is this argument used to criticize or defend the transitivity condition?

   The transitivity condition's first clause requires that if *a* P *b* and *b* P *c*, then *a* P *c*. The money pump argument is an argument about a problem with cyclical preferences, such as *a* P *b*, *b* P *c*, *c* P *a*. It claims that someone with such preferences is vulnerable to being "money pumped" because he or she would apparently be willing to engage in a series of trades in which he or she repeatedly pays to get a more-preferred item with the possible result that he or she will just end up with the same item that he or she started with, but less money. The money-pump argument is used to defend the transitivity condition by showing that it is disadvantageous to have cyclical preferences, which are the main example of preferences that violate the transitivity condition.

2. Suppose Louis has these preferences: *a* P *b*, *b* P *c*, *c* P *d*. And suppose Louis's preference for *a* over *b* is four times as strong as his preference for *b* over *c*, which in turn is one third as strong as his preference for *c* over *d*. What is an interval utility function that accurately represents Louis's preferences?

   There are infinitely many correct answers, but here is one: $u(a) = 8$, $u(b) = 4$, $u(c) = 3$, $u(d) = 0$.

3. Suppose option $A_1$ has a utility of 1 in state $S_1$, a utility of 2 in state $S_2$, and a utility of 3 in state $S_3$. Suppose option $A_2$ (in the same choice situation) has a utility of 4 in state $S_1$, 3 in state $S_2$, and 2 in state $S_3$. What, if anything, would the dominance principle recommend in this situation? What, if anything, would the maximin principle recommend in this situation? (Explain your reasoning for both principles.)

   The dominance principle has no recommendation, since there is no dominant option (since there is at least one state in which $A_1$ is inferior, and at least one state in which $A_2$ is inferior). The maximin principle would recommend $A_2$, since the minimum for it (2) is greater than the minimum for $A_1$ (1).

4. Suppose Greta is applying the maximax rule in a particular situation, for which she has written out a choice matrix. Do the numbers in the matrix need to be utilities from an interval utility function, or is it sufficient if they are from an ordinal utility function? Explain your answer.

   It is sufficient if they are from an ordinal utility function. The maximax rule only refers to the order in which the agent ranks the various possible outcomes, not the relative strengths of the person's preferences. Since the former information would be represented in an ordinal utility function, such a function would be sufficient.

For questions 5 and 6, let lottery L provide a 1/5 chance of winning \$1,000 and a 4/5 chance of winning \$0.

5. Suppose Betty prefers more money to less, but also prefers L to \$200. What are utility assignments for the three dollar amounts (\$0, \$200, and \$1,000) that make the principle of maximizing expected utility agree with Betty's preferences?

   Let $u(\$200) = u(\$0) + x$
   and $u(\$1,000) = u(\$200) + y = u(\$0) + x + y$
   (where $x$ and $y$ are both positive numbers)

   $$
   \begin{aligned}
   EU(L) &> EU(\$200)\\
   (1/5)u(\$1,000) + (4/5)u(\$0) &> u(\$200)\\
   u(\$1,000) + 4u(\$0) &> 5u(\$200)\\
   u(\$0) + x + y + 4u(\$0) &> 5[u(\$0) + x]\\
   5u(\$0) + x + y &> 5u(\$0) + 5x\\
   x + y &> 5x\\
   y &> 4x
   \end{aligned}
   $$

   There are infinitely many values for $x$ and $y$ that satisfy this constraint, but let's say that $x = 1$ and $y = 5$. Then we have $u(\$200) = u(\$0) + 1$, and $u(\$1,000) = u(\$0) + 1 + 5$. Now let's say that $u(\$0) = 0$. Then it follows that $u(\$200) = 1$ and $u(\$1,000) = 6$.

6. Suppose Wayne prefers \$300 to lottery L. Is he risk averse, risk neutral, or risk seeking, or do we not have enough information to say for sure?

   When a person prefers an amount of money to a lottery, we cannot infer that he is risk neutral or risk seeking; and we can infer that he is risk averse if and only if the lottery's expected monetary value is at least as great as the amount of money. But in this case, the lottery's expected monetary value is less than the amount of money, so we cannot infer that he is risk averse. We do not have enough information to say for sure.

For questions 7 and 8, assume that $u(\$50) = u(\$30) + x$, $u(\$60) = u(\$50) + y$, and $u(\$90) = u(\$60) + z$, with $x$, $y$, and $z$ being positive numbers. Also, the following instructions apply to both questions:

   a. Any equation or inequality in your answer should have just $z$ on the left, and just one mention of $x$ and/or one mention of $y$ on the right. Note that $x$ and $y$ may, of course, have coefficients; for example, it would be o.k. for the right side of your answer to be something like $-(1/5)x + 33y$.

   b. Show your work.

7. Suppose Penelope prefers a lottery giving her a 3/4 chance at \$60 and a 1/4 chance at \$50 to a lottery giving her a 3/4 chance at \$30 and a 1/4 chance at \$90. What constraint(s) on $x$, $y$, and $z$ (in addition to $x > 0$, $y > 0$, and $z > 0$) imply utility assignments for the four dollar amounts that make the principle of maximizing expected utility agree with this preference?

Let's start by establishing the following, which will be convenient later:

$u(\$50) = u(\$30) + x$
$u(\$60) = u(\$50) + y = u(\$30) + x + y$
$u(\$90) = u(\$60) + z = u(\$30) + x + y + z$

Now, let's work with the stated preference:

$$
\begin{aligned}
(3/4)u(\$60) + (1/4)u(\$50) &> (3/4)u(\$30) + (1/4)u(\$90) \\
3u(\$60) + u(\$50) &> 3u(\$30) + u(\$90) \\
3[u(\$30) + x + y] + u(\$30) + x &> 3u(\$30) + u(\$30) + x + y + z \\
3u(\$30) + 3x + 3y + u(\$30) + x &> 4u(\$30) + x + y + z \\
4x + 3y &> x + y + z \\
3x + 2y &> z \\
z &< 3x + 2y
\end{aligned}
$$

8. Suppose Penelope prefers \$60 to a lottery giving her a 2/3 chance at \$90 and a 1/3 chance at \$50. What constraint(s) on $x$, $y$, and $z$ (in addition to $x > 0$, $y > 0$, and $z > 0$) imply utility assignments for the four dollar amounts that make the principle of maximizing expected utility agree with this preference?

   Assuming the same formulas for $u(\$50)$, $u(\$60)$, and $u(\$90)$ that we derived for no. 7, we'll proceed to work with the stated preference:

$$
\begin{aligned}
u(\$60) &> (2/3)u(\$90) + (1/3)u(\$50) \\
3u(\$60) &> 2u(\$90) + u(\$50) \\
3[u(\$30) + x + y] &> 2[u(\$30) + x + y + z] + u(\$30) + x \\
3u(\$30) + 3x + 3y &> 2u(\$30) + 2x + 2y + 2z + u(\$30) + x \\
3x + 3y &> 3x + 2y + 2z \\
3y &> 2y + 2z \\
y &> 2z \\
(1/2)y &> z \\
z &< (1/2)y
\end{aligned}
$$

9. Do your answers to questions 7 and 8 imply that Penelope's preferences violate one or more of the conditions referred to in the representation theorem? Why or why not?

   No, they do not. In order for them to imply that Penelope's preferences violate one of more of those conditions, it would have to be impossible for utilities to be assigned to the items (the dollar amounts) over which she has preferences in a manner that makes her preferences compatible with the principle of maximizing expected utility. And in order for that to be the case, the conditions derived above would have to be incompatible. But they are not incompatible, since it is possible for to have $z < 3x + 2y$ and $z < (1/2)y$. For example, both conditions are satisfied when $x = 5$, $y = 5$, and $z = 1$.

10. What is one of the conditions referred to in the representation theorem? (You do not have to remember the name of any of the conditions but you to have to accurately state the content of one of the conditions.)

    (See pp. 175–180 of section 3.5 for descriptions of all of the conditions.)

***Instructions, revisited:***

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.