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Philosophy 666: Rational Choice Theory
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Test on Game Theory—Answer Key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period—no credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Special instruction:

Some of the matrices given below have only one number in each cell. That number should be regarded as the row player's payoff, with the column player's payoff being the negation of that number.

Questions:

1. Assume that game *G* is played by player 1 choosing $x = 1$, $x = 2$, or $x = 3$, then player 2 choosing $y = 11$, $y = 12$, $y = 13$, or $y = 14$, then player 1 choosing $z = 7$ or $z = 8$, then payoffs to both players being some function of *x*, *y*, and *z* (which is common knowledge between the players). How many rows does the matrix for *G* have, and how many columns does it have, if player 1 is the row player and player 2 is the column player?

   First, let us ascertain the number of rows by ascertaining the number of strategies for player 1. Any strategy for player 1 will begin with a specification for *x*, of which there are 3 possibilities. Then the strategy needs to have a plan for what to do in response to any of player 2's specifications for *y*. At that juncture player 1 has 2 options, and that juncture could come about in any of 4 different ways (based on player 2's having 4 different options for *y*). So, we have 2^4, or 16. Since this is independent of player 1's 3 different options for his or her specification of *x*, we have 3×16, or 48.

   Second, we'll ascertain the number of columns by ascertaining the number of strategies for player 2. A strategy for player 2 consists of a plan for what to do in response to any of player 1's specifications for *x*. At that juncture player 2 has 4 options, and that juncture could come about in any of 3 different ways (based on player 1's having 3 different options for *x*). So, we have 4^3, or 64.

2. Analyze the following game using dominance considerations and write the strategy pair(s) corresponding to its solution(s). Write each strategy pair in the form (R_x, C_y), where *x* and *y* are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3	C_4
R_1	5	4	4	6
R_2	5	6	9	2
R_3	7	6	8	7

   (R_3, C_2)

3. State whether the following game has any equilibrium strategy pair(s). (You can ignore mixed strategies and focus on pure strategies only.) If it does, write it (or each of them) in the form (R_x, C_y), where x and y are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3	C_4
R_1	5	3	3	4
R_2	8	5	6	5
R_3	5	5	7	5

Yes—(R_2, C_2), (R_2, C_4), (R_3, C_2), and (R_3, C_4).

4. Imagine a two-person zero-sum game in which both players have two pure strategies. Suppose that when the row player plays a mixed strategy of the form (p R_1, $(1 - p)$ R_2) against a mixed strategy for the column player of the form (q C_1, $(1 - q)$ C_2), the row player's expected utility is $[p \times (15q - 5)] + (-4q + 7)$. What mixed strategy should the column player play, if she would like to play a mixed strategy that could (along with a correctly chosen mixed strategy for the row player) be part of an equilibrium strategy pair? (Be sure to write a mixed strategy for the column player, not just the value of a variable.)

The column player should play the mixed strategy corresponding to the value of q that makes the row player's expected utility independent of the value of p. Given that the expression for the row player's expected utility has p multiplied by $15q - 5$, this latter expression should be set equal to 0. That yields the equation $15q - 5 = 0$, or $15q = 5$, or $q = 5/15$, or $q = 1/3$. So, the column player should play the mixed strategy (1/3 C_1, 2/3 C_2).

5. What values of p and q make (p R_1, $(1 - p)$ R_2; q C_1, $(1 - q)$ C_2) an equilibrium strategy pair for the following game? (You do not have to show your work. An answer of the form 'p = __, q = __' can earn full credit.)

	C_1	C_2
R_1	9	3
R_2	2	6

$$p = \frac{6-2}{(6-2)+(9-3)} = \frac{4}{4+6} = \frac{4}{10} = \frac{2}{5}$$

$$q = \frac{6-3}{(6-3)+(9-2)} = \frac{3}{3+7} = \frac{3}{10}$$

6. Imagine a two-person zero-sum game in which both players have two pure strategies, and in which it is known to both players that the row player is playing the mixed strategy (1/2 R_1, 1/2 R_2). The column player realizes that if she also plays a mixed strategy giving equal probabilities to each of her pure strategies (that is, if she plays the mixed strategy (1/2 C_1, 1/2 C_2), then her expected utility is as large as it would be with any other mixed strategy that she might choose to play (assuming, throughout, the mixed strategy (1/2 R_1, 1/2 R_2) for the row player). Does this mean that [(1/2 R_1, 1/2 R_2), (1/2 C_1, 1/2 C_2)] is an equilibrium strategy pair for this game? Why or why not?

No, it does not. We know that if the row player is playing (1/2 R_1, 1/2 R_2) and the column player is playing (1/2 C_1, 1/2 C_2), the column player has no incentive to unilaterally deviate. But we do not know the analogous fact about the row player; given that the column player is playing (1/2 C_1, 1/2 C_2), the row player might have an incentive to play some other strategy than (1/2 R_1, 1/2 R_2). Given this possibility, we cannot conclude that [(1/2 R_1, 1/2 R_2), (1/2 C_1, 1/2 C_2)] is an equilibrium strategy pair for this game.

7. Suppose that, in the following game, the row player knows that the column player is going to play the mixed strategy (3/4 C_1, 1/4 C_2). Of all of the row player's pure and mixed strategies, which one has the greatest expected utility for him?

	C_1	C_2
R_1	3	8
R_2	7	2

All of the row player's pure and mixed strategies are included in the general form (p R_1, $(1 - p)$ R_2). So, we can see what the expected utility of (p R_1, $(1 - p)$ R_2) is, on the assumption that the column player is playing the mixed strategy (3/4 C_1, 1/4 C_2), and see what value of p will maximize the value of that expression.

$EU(p\ R_1, (1 - p)\ R_2)$
$= (p)EU(R_1) + (1 - p)EU(R_2)$
$= (p)[(3/4)(3) + (1/4)(8)] + (1 - p)[(3/4)(7) + (1/4)(2)]$
$= (p)(9/4 + 8/4) + (1 - p)(21/4 + 2/4)$
$= (p)(17/4) + (1 - p)(23/4)$
$= 17p/4 + 23/4 - 23p/4$
$= -6p/4 + 23/4$

Since the expected utility is negatively influenced by p, the player should make p as small as possible. Since $0 \leq p \leq 1$, the minimum possible value for p is 0. So, the player should play the mixed strategy (0 R_1, 1 R_2), which of course reduces to the pure strategy R_2.

8. State whether the following game has any equilibrium strategy pair(s). (You can ignore mixed strategies and focus on pure strategies only.) If it does, write it (or each of them) in the form (R_x, C_y), where x and y are integers corresponding to row and column numbers, respectively. Is this game a coordination game (also known as a battle of wills), a prisoner's dilemma, or neither?

	C_1	C_2
R_1	3, 3	8, 4
R_2	4, 7	1, 2

Yes—(R_1, C_2) and (R_2, C_1). This game is a coordination game.

9. Follow the instructions given for the previous question, but for the following game.

	C_1	C_2
R_1	4, 8	9, 8
R_2	1, 5	9, 4

Yes—(R_1, C_1) and (R_1, C_2). This game is neither a coordination game nor a prisoner's dilemma.

10. This question consists of two multiple-choice questions.

10.1 Which of the following statements is true of coordination games?

A. Every equilibrium outcome is Pareto optimal.

B. Some equilibrium outcomes are Pareto optimal, and some are not.

C. No equilibrium outcome is Pareto optimal.

A. (Every equilibrium outcome is Pareto optimal. There are two, and each is the uniquely most-preferred outcome of one player or the other, making each Pareto optimal.)

10.2 Which of the following statements is true of prisoner's dilemmas?

(same answers as for question 10.1)

C. (No equilibrium outcome is Pareto optimal. There is just one, and there is another outcome that would be better for both players.)

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.