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Philosophy 666: Rational Choice Theory
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Test on Social Choice Theory—Answer Key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.

2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period—no credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.

3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Special instruction:

The following two profiles and corresponding social preference orderings are referred to in some of the questions given below. Assume that the social preference orderings were generated by some social welfare function called 'F'.

Profile 1:			s.p.o.	Profile 2:			s.p.o.
A	B	C		A	B	C	
a	b	c	c	a	b	d	a
b	a, d	d	a	b	a, d	c	c
c	c	b	d	c	c	b	d
d		a	b	d		a	b

Questions:

1. *Borda count:*

1a. What is the social preference ordering that the Borda count would generate for profile 1?

alternative	points from A	points from B	points from C	total points
a	4	2.5	1	7.5
b	3	4	2	9
c	2	1	4	7
d	1	2.5	3	6.5
[totals:]	10	10	10	30

Resulting social preference ordering:

b
a
c
d

1b. What is a condition on social welfare functions that the Borda count violates?

Condition I (independence of irrelevant alternatives).

2.   *Pairwise majority rule:*

2a. What is the social preference ordering that pairwise majority rule would generate for profile 2?

pair	number preferring first alternative	number preferring second alternative	social preference
a vs. *b*	1	2	*b* P *a*
a vs. *c*	2	1	*a* P *c*
a vs. *d*	1	1	*a* I *d*
b vs. *c*	2	1	*b* P *c*
b vs. *d*	2	1	*b* P *d*
c vs. *d*	1	2	*d* P *c*

Resulting social preference ordering:

 b
a, d
 c

2b. What is a condition on social welfare functions that pairwise majority rule violates?

Condition U (unrestricted domain).

3.   *Pairwise majority rule, continued:*

3a. Can either profile 1 or profile 2 be used to show that pairwise majority rule violates the condition you just mentioned (in your answer to question 2b)? Why or why not?

The answer to question 2a establishes that profile 2 cannot be so used, since that answer shows that the social preferences that pairwise majority rule determines for that profile can be put into a social preference ordering. But profile 1 can be so used, as shown by the following table:

pair	number preferring first alternative	number preferring second alternative	social preference
a vs. *b*	1	2	*b* P *a*
a vs. *c*	2	1	*a* P *c*
a vs. *d*	1	1	*a* I *d*
b vs. *c*	2	1	*b* P *c*
b vs. *d*	2	1	*b* P *d*
c vs. *d*	2	1	*c* P *d*

This table includes the following preferences, which are cyclical:

a P *c*, *c* P *d*, *d* I *a*

3b. *This sub-question is optional. If you answered question 3a correctly, you have already earned full credit for question 3. But if you are concerned that you might not have answered question 3a correctly and would like to guarantee yourself at least half credit for question 3, you can answer the following question:* What is an example of a profile that shows that pairwise majority rule violates a condition on social welfare functions?

A	B	C
a	*c*	*b*
b	*a*	*c*
c	*b*	*a*

(This profile shows that pairwise majority rule violates condition U.)

4. *Instant runoff voting:*

4a. What is the social preference ordering that instant runoff voting would generate for the following profile?

A, B	C, D, E	F, G, H	I, J, K, L
a	*b*	*c*	*d*
b	*a, d*	*d*	*a*
c	*c*	*b*	*b*
d		*a*	*c*

First *a* is eliminated, then *c*, then *b*, leaving *d*. Resulting social preference ordering:

d
b
c
a

4b. What is a condition on social welfare functions that instant runoff voting violates?

Condition M (monotonicity); also condition I (independence of irrelevant alternatives).

5. Which one of the following statements is true?

   a. Profiles 1 and 2 and their corresponding F-determined social preference orderings entail that F satisfies condition I.

   b. Profiles 1 and 2 and their corresponding F-determined social preference orderings entail that F violates condition I.

   c. Profiles 1 and 2 and their corresponding F-determined social preference orderings do not entail either that F satisfies condition I or that F violates condition I.

   b. (In the social preference orderings, the relative positions of alternatives *a* and *c* are reversed even though no individual changes his or her relative ranking of those two alternatives between profile 1 and profile 2.)

6. Same question as no. 5, but replace 'condition I' with 'condition M' in the three answer choices.

   c. (In these profiles, there is no alternative that moves up in one or more individual preference orderings but moves down in the social preference ordering. So, the answer is not b, and more information would be needed in order for the answer to be a.)

7. Can it be ascertained from profiles 1 and 2 and their corresponding F-determined social preference ordering whether F satisfies or violates condition NI? If so, how? If not, what additional information is needed?

    No, this cannot be ascertained. The additional information needed is whether *every* preference ordering is the F-determined social preference ordering corresponding to some profile or other. Profiles 1 and 2 just show that *two* preference orderings are among the F-determined social preference orderings corresponding to some profiles.

8. Same question as no. 7, but replace 'condition NI' with 'condition ND'.

    Yes, this can be ascertained. The F-determined social preference ordering corresponding to profile 1 is sufficient to show that F satisfies condition ND, since that social preference ordering is different from the individual preference ordering of every person in that profile, meaning that no person is a dictator.

9. What does Arrow's impossibility theorem say?

    If the number of alternatives is at least 3 and the number of people is at least 2, no social welfare function satisfies conditions U, I, M, NI, and ND.

10. Suppose that social welfare function G satisfies conditions P and L, person 1 has control (in the condition-L sense) over *a* versus *b*, and person 2 has control (in the condition-L sense) over *a* versus *c*. (As usual, assume that *a*, *b*, and *c* are distinct from each other, rather than there being any alternative that is referred to using multiple names.) Give an example of a profile that shows that G does not satisfy condition U.

    <u>1</u>   <u>2</u>

    a   *b*
    b   *c*
    c   *a*

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.