

## Test on Utility Theory – Answer Key (#4, 7, 9, 10)

**Instructions:**

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

**Questions:**

1. Suppose there are three items –  $a$ ,  $b$ , and  $c$  – and a person has the preferences  $a P b$  and  $a P c$ . Suppose the person says, “My preferences satisfy the completeness condition because every option is mentioned in at least one of my preferences.” Does this remark reflect a correct understanding of the completeness condition? Why or why not?
2. Consider the following statement: “The transitivity condition requires that if a person prefers  $a$  to  $b$ , and  $b$  to  $c$ , then the person must also prefer  $a$  to  $c$ . One cost of having transitive preferences is that a person can be used as a money pump. For example, if the person just mentioned possesses item  $c$ , his or her preference  $a P c$  (which is required by the transitivity condition) means he or she must be willing to pay some amount of money to exchange it (item  $c$ ) for item  $a$ .” Does this remark express the money-pump argument? Why or why not?
3. Suppose Christine has these preferences:  $a P b$ ,  $b P c$ ,  $c P d$ . And suppose Christine’s preference for  $a$  over  $b$  is one half as strong as her preference for  $b$  over  $c$ , which in turn is one fifth as strong as her preference for  $c$  over  $d$ . What is an interval utility function that accurately represents Christine’s preferences?
4. Suppose a defendant whose trial has just gone to the jury is deciding whether to accept a last-minute plea bargain from the prosecutor. The defendant can either accept the prosecutor’s offer or reject it. If he rejects the offer, he will face one of the following three judgments from the jury: (1) innocent, (2) guilty and mild punishment, and (3) guilty and severe punishment. If he accepts the offer, he will get the second of the outcomes just mentioned: a guilty verdict and a mild punishment. Write the choice matrix for this situation.

	jury says innocent	jury says guilty and mild punishment	jury says guilty and severe punishment
accept offer	guilty verdict and mild punishment	guilty verdict and mild punishment	guilty verdict and mild punishment
reject offer	innocent verdict	guilty verdict and mild punishment	guilty verdict and severe punishment

5. What is the maximin rule? What are the main reasons that can be given for and against it?

(questions continued on back)

The following choice matrix is for questions 6–8.

	S <sub>1</sub>	S <sub>2</sub>
A <sub>1</sub>	8	4
A <sub>2</sub>	1	x

- Suppose Frank believes that in state S<sub>2</sub>, option A<sub>2</sub> will have a utility of 10. (That is, in Frank’s opinion,  $x = 10$ .) And suppose Frank decides to choose between options A<sub>1</sub> and A<sub>2</sub> using the optimism-pessimism rule, with an optimism index of 1/3. Which option would the rule recommend? Show your work.
- Suppose Gretchen also believes  $x = 10$ , and decides to choose between options A<sub>1</sub> and A<sub>2</sub> using the minimax regret rule. Which option would the rule recommend? Show your work.

	S <sub>1</sub>	S <sub>2</sub>	<u>maximum regret for each option:</u>		
regret matrix:	A <sub>1</sub>	<table border="1"><tr><td>0</td><td>6</td></tr></table>	0	6	Ⓔ
0	6				
	A <sub>2</sub>	<table border="1"><tr><td>7</td><td>0</td></tr></table>	7	0	7
7	0				

The smallest maximum regret is circled. It is in the row for A<sub>1</sub>. So, A<sub>1</sub> is what the rule would recommend.

- Suppose Harry decides to choose between options A<sub>1</sub> and A<sub>2</sub> using the rule of maximizing expected utility using the principle of insufficient reason. If Harry applies the rule correctly and finds that it recommends A<sub>2</sub>, what must he believe about  $x$ ? (That is, what inequality of the form  $x > \underline{\quad}$  or  $x < \underline{\quad}$  must Harry believe is true?)

For questions 9 and 10, let L be a lottery that provides a 1/4 chance of winning \$1,000 and a 3/4 chance of winning \$0. Also, assume the following:

$$u(\$200) = u(\$0) + x$$

$$u(\$1,000) = u(\$200) + y$$

$$x > 0$$

$$y > 0$$

9. Suppose Iris prefers \$200 to L. What constraint concerning  $x$  and  $y$  (in addition to the constraints just stated) implies utility assignments for the three dollar amounts (\$0, \$200, and \$1,000) that make the principle of maximizing expected utility agree with this preference?

$$EU(\$200) > EU(L)$$

$$u(\$200) > (1/4)(u(\$1,000)) + (3/4)u(\$0)$$

$$4u(\$200) > u(\$1,000) + 3u(\$0)$$

$$4[u(\$0) + x] > u(\$0) + x + y + 3u(\$0)$$

$$4u(\$0) + 4x > 4u(\$0) + x + y$$

$$4x > x + y$$

$$3x > y$$

10. What are utility assignments for the three dollar amounts (\$0, \$200, and \$1,000) that make the principle of maximizing expected utility agree with this preference?

$$u(\$0) = 0$$

$$u(\$200) = 1$$

$$u(\$1,000) = 2$$

***Instructions, revisited:***

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.