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University of Kansas, Spring 2014
Philosophy 666: Rational Choice Theory

Questions for Credit

rules:

1. You can get help and advice from any source, as long as you fully comprehend the answers you turn in.
2. Answers are due in writing at the beginning of class unless otherwise specified.
3. If you are going to miss class, you can drop your answers off at my office or submit them by e-mail, but you must turn them in by the time class starts.
4. Whenever possible, please submit your answers in hard copy rather than by e-mail.

questions due Monday, January 27 (course overview):

1. Informally give an example of a choice situation that would most appropriately be handled by individual decision theory. (It does not have to be different from the situation you thought of for class on Friday, January 24.)

questions due Wednesday, January 29 (introduction and section 1.1):

2. What are the two main things rational choice theory is concerned with?

3. Represent a choice situation using an outline of the kind discussed in section 1.1. (It does not have to be different from the choice situation you informally described in answer to question 1.)

questions due Friday, January 31 (sections 1.2–1.3):

4. Represent a choice situation using an outline of the kind discussed in section 1.1, and add to it by attaching probabilities to some or all of the possible outcomes. State whether the choice situation thus represented is one of certainty, uncertainty, or risk.

questions due Monday, February 3 (sections 2.1–2.2):

5. Write a paragraph in which some preferences over at least four items are stated in ordinary conversational English. Then re-state that same information formally, using ‘P’, ‘I’, abbreviations or short names for the items, and/or a glossary, as needed.

6. Suppose a person considers options {a, b, c} and forms the following preferences: *a* P *b*, *b* P *c*. Do the person’s preferences satisfy the completeness condition?

7. Suppose Jenny reports that she has no preference between *c* and *d*—not even the “preference” of indifference. That is, suppose Jenny does not affirm *c* P *d*, or *d* P *c*, or *c* I *d*. Suppose also that Karl, a defender of the completeness condition, says that if Jenny really does not affirm *c* P *d* or *d* P *c*, then she must really affirm *c* I *d*, despite her report to the contrary. Has Karl just used the small-improvement argument? Explain your answer.

questions due Friday, February 7 (section 2.3):

8. Consider the following preferences: *j* P *k*, *k* P *l*, *l* P *m*, *m* P *n*, *n* P *j*. Prove that these preferences violate the transitivity condition.

9. Give an example in which a person has preferences that make him or her vulnerable to being used as a money pump.

questions due Monday, February 10 (section 2.4):

10. Evaluate the following preferences according to the instructions for the practice questions in section 2.4.

    preferences:

    1. e P a
    2. g P e
    3. b P c
    4. i P g
    5. a P b
    6. a P d
    7. c P e
    8. d P f

questions due Wednesday, February 12 (section 3.1):

11. Why does the following table not represent a utility function?

x	$u(x)$
blue, green	8
orange	5
red	4, 3

questions due Friday, February 14 (section 3.2):

12. Provide an example of an interval measurement scale. The example you provide can be either an established measurement scale, or just a way of recording some information in a specific situation that you describe, but it must not be any interval measurement scale discussed in section 3.2 (whether in the main text or in the practice questions).

13. Consider the following preference ordering:

    a
    b
    c
    d

    Suppose the preference for a over b is one fifth as strong as the preference for b over c, which in turn is four times as strong as the preference for c over d. Does the following utility function represent these preferences? Why or why not?

x	$u(x)$
a	6
b	5
c	0
d	–4

14. Provide an interval utility function that represents the preferences stated in question 13.

15. What is the utility function that results from applying the transformation $u'(x) = -2u(x) + 50$ to the utility function you provided in answer to question 14? Does it represent the preferences stated in question 13? Why or why not?

questions due Monday, February 17 (sections 4.1–4.4):

16. You have time in the near future to read either a philosophy book or a biography. If you read the philosophy book, you will be either delighted or really annoyed; and if you read the biography, you will be either mildly pleased or mildly disappointed. Create a matrix for this situation.

The following matrix is for questions 17–19.

	S_1	S_2	S_3
A_1	4	6	2
A_2	5	3	x

17. Are there values of x that make option A_1 uniquely selected by the dominance principle? If so, what inequality describes those values of x?

18. Are there values of x that make option A_1 uniquely selected by the maximin rule? If so, what inequality describes those values of x?

19. Are there values of x that make option A_1 uniquely selected by the maximax rule? If so, what inequality describes those values of x?

questions due Wednesday, February 19 (sections 4.5–4.6):

The following matrix is for questions 20–23.

	S_1	S_2	S_3	S_4
A_1	10	x	2	6
A_2	4	9	8	5
A_3	11	1	3	7

20. What must be true of x in order for option A_1 to be uniquely selected by the optimism-pessimism rule if $\alpha = 2/5$? (Hint: this involves computing the α-index of the other two options, and then figuring out what must be true of x in order for the α-index of A_1 to be the largest of the three α-indexes.)

21. What is the regret matrix that follows from this utility matrix, if $x > 9$?

questions due Friday, February 21 (sections 4.6–4.8):

22. What must be true of x in order for option A_1 to be uniquely selected by the minimax regret rule?

23. What must be true of x in order for option A_1 to be uniquely selected by the approach of maximizing expected utility using the principle of insufficient reason?

questions due Monday, February 24 (chapter 5):

24. In the situation represented by the following matrix, what must be true of x in order for A_1 to have a higher expected utility than A_2?

	S_1 (2/3)	S_2 (1/3)
A_1	7	x
A_2	4	9

25. Answer the same question as the previous one, but with the probabilities of states S_1 and S_2 reversed (1/3 and 2/3, respectively, rather than 2/3 and 1/3).

26. Suppose you are a police chief hoping to deter the commission of a type of crime by making perpetrators' chances of being apprehended and convicted sufficiently high. If not committing the crime has a utility of 0, committing the crime has a utility of 3 (enjoyed by the criminal whether subsequently punished or not), and being punished has a utility of –8, how likely must apprehension and conviction be, in order for the punishment to be a sufficient deterrent?

27. Answer the same question as the previous one, but with these new assumptions: not committing the crime has a utility of 2, committing the crime has a utility of 5 (enjoyed by the criminal whether subsequently punished or not), and being punished has a utility of –6.

questions due Wednesday, February 26 (section 6.1):

For questions 28–30, assume the following:

L_1 is a lottery with a 1/4 chance at \$100 and 3/4 chance at \$5.

L_2 is a lottery with a 1/2 chance at \$50 and a 1/2 chance at \$0.

$u(\$5) = u(\$0) + x$

$u(\$50) = u(\$5) + y$

$u(\$100) = u(\$50) + z$

28. Suppose that more money is preferred to less, and that L_2 is preferred to L_1. What constraint concerning x, y, and z can be derived from these suppositions? (You can regard the constraints $x > 0$, $y > 0$, and $z > 0$ as assumed rather than stating them.)

29. State values for x, y, and z that satisfy the constraint you stated in answer to the previous question.

30. Show that if $u(\$0) = 0$, and the utilities of \$5, \$50, and \$100 are assigned in accordance with the values of x, y, and z you stated in answer to the previous question, then $EU(L_2) > EU(L_1)$.

questions due Wednesday, March 5:

For questions 31–34, assume that game *G* is played by player 1 choosing $x = 1$, $x = 2$, or $x = 3$, then player 2 (with knowledge of player 1's choice) choosing $y = 4$ or $y = 5$, then payoffs to both players being some function of x and y (which is common knowledge between the players).

31. Draw the tree for this game; make it approximately the size of one letter-sized sheet of paper. (You do not have to label the ends of the "branches," since you are not told the details of the payoffs.)

32. List player 1's strategies for this game.

33. List player 2's strategies for this game. (Hint: Here is one of them; be sure to include it in your list. "If $x = 1$, choose $y = 4$; if $x = 2$, choose $y = 5$; if $x = 3$, choose $y = 4$.")

34. If a matrix for this game had player 1 as the row player and player 2 as the column player, how many rows would the matrix have, and how many columns would it have?

questions due Friday, March 7:

35. Analyze the following game using dominance considerations and write the strategy pair(s) corresponding to its solution(s). (For example, you might write '(R_1, C_1), (R_3, C_3)' – but that is not the right answer.) In the matrix, the numbers in the outcome cells are the utilities for the row player; assume that, for all outcomes, the column player's utilities are the negations of the row player's utilities.

	C_1	C_2	C_3
R_1	7	3	2
R_2	4	2	9
R_3	9	6	7

36. The matrix in question 35 is one in which (1) each player has at least three strategies, (2) neither player has a dominant strategy, (3) dominance considerations eliminate more than half of the outcomes, and (4) only the row player's utilities are stated because the column player's utilities are understood to be the negations of the row player's utilities. Write another matrix that has these four characteristics, and write the strategy pair(s) corresponding to its solution(s).

questions due Monday, March 10:

For questions 37–39, state whether the game has any equilibrium strategy pair(s). If it does, write it (or them).

37.

	C_1	C_2	C_3
R_1	4	7	9
R_2	4	6	9

38.

	C_1	C_2	C_3
R_1	1	3	6
R_2	3	9	2
R_3	2	6	8

39.

	C_1	C_2	C_3	C_4
R_1	8	5	7	5
R_2	1	1	9	3
R_3	6	5	8	5

questions due Wednesday, March 12:

Use the following game for questions 40–45.

	C_1 (q)	C_2 ($1 - q$)
R_1 (p)	3	9
R_2 ($1 - p$)	7	4

40. Suppose $q = 1/6$. What is the expected value (for the row player) of playing strategy R_1, and what is the expected value of playing strategy R_2?

41. Suppose $q = 1/6$. What is the expected value (for the row player) of playing strategy (p R_1, ($1 - p$) R_2)?

42. Is it possible for the column player's mixed strategy (1/6 C_1, 5/6 C_2) to be one half of a pair of mixed strategies (one mixed strategy for the row player, one mixed strategy for the column player) that is in equilibrium? Why or why not?

43. Suppose $q = 5/9$. What is the expected value (for the row player) of playing strategy R_1, and what is the expected value of playing strategy R_2?

44. Suppose $q = 5/9$. What is the expected value (for the row player) of playing strategy (p R_1, ($1 - p$) R_2)?

45. Is it possible for the column player's mixed strategy (5/9 C_1, 4/9 C_2) to be one half of a pair of mixed strategies (one mixed strategy for the row player, one mixed strategy for the column player) that is in equilibrium? Why or why not?

questions due Friday, March 14:

Use the following game for questions 46–47. Remember that when you are asked to show that two things are equal, you should compute each of them separately and then point out that the computed values are the same – do not just start with an equation stating that the two things are equal.

	C_1 (q)	C_2 ($1 - q$)
R_1 (p)	4	44
R_2 ($1 - p$)	55	5

46. Show that if the row player sets p equal to 50/90, then (for the column player) $EU(C_1) = EU(C_2)$.
(You can just reduce the 50/90 to 5/9 and work with that. However, it will be useful later to think of it as 50/90.)

47. Show that if the column player sets q equal to 39/90, then (for the row player) $EU(R_1) = EU(R_2)$.

questions due Monday, March 24:

Use the following game for questions 48–49.

	C_1 (q)	C_2 $(1 - q)$
R_1 (p)	13	4
R_2 $(1 - p)$	7	11

48. Derive the value of p that makes (p R_1, $(1 - p)$ R_2) the row player's half of an equilibrium strategy pair. Start with either the equation $EU(R_1) = EU(R_2)$ or the equation $EU(C_1) = EU(C_2)$ – whichever is appropriate – and show your work.

49. Derive the value of q that makes (q C_1, $(1 - q)$ C_2) the column player's half of an equilibrium strategy pair. Start with either the equation $EU(R_1) = EU(R_2)$ or the equation $EU(C_1) = EU(C_2)$ – whichever is appropriate – and show your work.

Use the following game for questions 50–51.

	C_1 (q)	C_2 $(1 - q)$
R_1 (p)	2	8
R_2 $(1 - p)$	7	4

50. What is the value of p that makes (p R_1, $(1 - p)$ R_2) the row player's half of an equilibrium strategy pair? (You do not have to show your work.)

51. What is the value of q that makes (q C_1, $(1 - q)$ C_2) the column player's half of an equilibrium strategy pair? (You do not have to show your work.)

questions due Wednesday, March 26:

Use the following game for questions 52–54. For all three questions, assume that the row player believes that the column player will choose her strategy (C_1 or C_2) by flipping a fair coin.

	C_1 (q)	C_2 $(1 - q)$
R_1 (p)	9	4
R_2 $(1 - p)$	6	8

52. What is the expected utility (for the row player) of R_1, and what is the expected utility of R_2? Which strategy's expected utility is greater?

53. What is the expected utility (for the row player) of his general strategy (p R_1, $(1 - p)$ R_2)? (Your answer should be some expression involving the variable p.)

54. Based on your answer to question 53, what should the row player make the value of p be, in order to maximize his expected utility of playing the game? How does your answer to this question confirm part(s) of your answer to question 52?

questions due Friday, March 28:

55. Is the following game a coordination game? Why or why not?

	C_1	C_2
R_1	7, 10	10, 9
R_2	9, 4	5, 5

56. Is the following game a coordination game? Why or why not?

	C_1	C_2
R_1	8, 7	5, 4
R_2	6, 5	11, 8

57. Is the following game a coordination game? Why or why not?

	C_1	C_2
R_1	3, 3	8, 9
R_2	10, 7	6, 6

questions due Monday, March 31:

58. Is the following game a prisoner's dilemma? Why or why not?

	C_1	C_2
R_1	0, 30	8, 20
R_2	2, 15	40, 12

59. Is the following game a prisoner's dilemma? Why or why not?

	C_1	C_2
R_1	8, 5	–2, –2
R_2	–3, –1	6, 7

60. Is the following game a prisoner's dilemma? Why or why not?

	C_1	C_2
R_1	8, 2	11, 1
R_2	9, 0	8, 6

questions due Wednesday, April 2:

61. Is (or are) any of the outcomes of the following game Pareto optimal? If so, which one(s)?

	C_1	C_2
R_1	9, 3	6, 4
R_2	6, 8	7, 2

62. Consider applying the model of p. 38 to the following prisoner's dilemma. What should we take the values of *u* and *u*' to be, in order to apply that model?

	C_1	C_2
R_1	2/5, 2/5	0, 1
R_2	1, 0	1/3, 1/3

questions due Wednesday, April 9 (sections 1–3):

63. How many profiles can be constructed from three alternatives and five people?

64. Suppose you are presented with the following profile. How many other profiles can be constructed from the same number of alternatives and the same number of people?

A	B	C	D
a	*b*	*a*, *b*	*b*
b	*a*		*a*

65. Write out either (a) all of the other profiles that can be constructed from the same number of alternatives and the same number of people as the one given in the previous question or (b) all of the profiles that can be constructed from two alternatives and two people.

questions due Friday, April 11 (sections 4–5):

For questions 66–68, let F be a social welfare function, and consider the following profiles and F-determined corresponding social preference orderings:

1:

A	B	C	soc.
c	*b*	*c*	*b*
d	*a*	*b*	*a*
b	*d*	*a*	*c*
a	*c*	*d*	*d*

2:

A	B	C	soc.
b	*b*	*a*	*b*
d	*c*	*d*	*d*
c	*a*	*b*	*c*
a	*d*	*c*	*a*

66. If we look only at the positions of alternatives *a* and *d*, do these profiles and corresponding social preference orderings prove that F violates condition I? How about if we look only at the positions of alternatives *b* and *c*?

67. If we look only at the positions of alternatives *a* and *c*, do these profiles and corresponding social preference orderings prove that F violates condition I? How about if we look only at the positions of alternatives *b* and *d*?

68. Are there any other pairs of alternatives that one would need to check (in order to continue to test F for compliance with condition I), if the answers to questions 66 and 67 were all negative? If so, what are they?

questions due Monday, April 14 (sections 6–7):

69. For the following profile, figure out the social preference ordering determined by pairwise majority rule. If there is not one, show why.

A	B	C	D
a, *b*, *c*	*a*	*b*	*a*
d	*b*	*c*, *d*	*b*
	d	*a*	*c*, *d*
	c		

70. Same instructions as for question 69, but in regard to the following profile:

A	B	C	D
a	*a*, *c*	*b*	*c*
b	*d*	*d*	*a*, *d*
c	*b*	*a*	*b*
d		*c*	

71. What can be concluded, strictly from question 69 and your answer to it, about whether pairwise majority rule satisfies condition U? How about from question 70 and your answer to it?

questions due Wednesday, April 16 (sections 8–10):

The following profile is for questions 72–73.

A	B	C	D	E
d	*c*	*a*	*d*	*c*
b, *c*	*a*	*b*	*c*	*b*, *d*
a	*b*	*c*, *d*	*a*	*a*
	d		*b*	

72. What social preference ordering would be dictated by plurality voting?

73. What social preference ordering would be dictated by instant runoff voting?

For questions 74–75, let F be a social welfare function, and consider the following profiles and F-determined corresponding social preference orderings:

1:

A	B	C	soc.
c	*b*	*a*	*b*
a	*d*	*c*	*a*
d	*c*	*b*	*c*
b	*a*	*d*	*d*

2:

A	B	C	soc.
c	*b*	*a*	*b*
a	*c*	*c*	*a*
d	*d*	*b*	*d*
b	*a*	*d*	*c*

3:

A	B	C	soc.
b	*b*	*d*	*b*
a	*c*	*c*	*c*
c	*d*	*a*	*a*
d	*a*	*b*	*d*

4:

A	B	C	soc.
a	*b*	*d*	*c*
b	*c*	*c*	*b*
c	*d*	*a*	*a*
d	*a*	*b*	*d*

74. Do profiles 1 and 3 and their corresponding social preference orderings prove that F violates condition M?

75. Do profiles 2 and 4 and their corresponding social preference orderings prove that F violates condition M?

questions due Friday, April 18 (sections 11–12):

76. When there are 3 alternatives, there are 13 preference orderings, so when there are 3 alternatives and 2 people, there are 13^2, or 169, profiles. Suppose these profiles are put in an order (any order) and numbered from 1 to 169. Then suppose F is a social welfare function that assigns "*a*; then *b* and *c* tied" as the social preference ordering for the first 160 of those 169 profiles. Can we conclude, from just the foregoing information, that F violates condition NI? Why or why not?

77. When there are 3 alternatives, there are 13 preference orderings, so when there are 3 alternatives and 5 people, there are 13^5, or 371,293, profiles. Suppose these profiles are put in an order (any order) and numbered from 1 to 371,293. Then suppose F is a social welfare function that makes the social preference ordering for each of the first 371,290 of these profiles the same as the individual preference ordering for person A in each of those profiles, respectively. Can we conclude, from just the foregoing information, that F violates condition ND? Why or why not?

questions due Monday, April 21 (section 13):

78. What preference(s) does condition P require a social welfare function to include in whatever social preference ordering it determines for the following profile?

A	B	C	D	E
c	*d*	*c*	*d*	*c*
a	*b*, *c*	*b*	*c*	*b*, *d*
b	*a*	*a*, *d*	*a*	*a*
d			*b*	

questions due Wednesday, April 23 (sections 14–15):

79. Is it possible for two profiles consisting of three alternatives and two people, and their corresponding F-determined social preference orderings, to prove that F violates condition M without also proving that F violates condition I? If so, give an example of two such profiles and their corresponding social preference orderings.

80. Is it possible for two profiles consisting of three alternatives and two people, and their corresponding F-determined social preference orderings, to prove that F violates both condition M and condition NI? If so, give an example of two such profiles and their corresponding social preference orderings.

questions due Friday, April 25 (sections 15–16):

81. Is every Paretian social welfare function dictatorial? Is every dictatorial social welfare function Paretian?

82. Suppose you were trying to prove a statement concerning how a particular social welfare function F would rank alternative *a* in the social preference ordering corresponding to a particular profile P1. And suppose you knew the following: (1) another profile, P2, is just like P1 except that one of the individuals ranks alternative *a* lower in P2 than in P1 and (2) F specifies a social preference ordering for P2 that has alternative *a* in first place. If you wanted to prove that F specifies a social preference ordering for P1 that has alternative *a* in first place, is there a particular condition you could appeal to, in order to prove that? (That is, is there a particular condition such that, if F satisfies that condition, then the desired statement follows?) If so, which one?

83. Suppose you were trying to prove a statement concerning how a particular social welfare function F would rank alternative *a*, relative to alternative *b*, in the social preference ordering corresponding to a particular profile P. And suppose you knew that, in P, every individual ranks *b* above *a*. If you wanted to prove that F specifies a social preference ordering for P that has *b* ranked above *a*, is there a particular condition you could appeal to, in order to prove that? (That is, is there a particular condition such that, if F satisfies that condition, then the desired statement follows?) If so, which one?

84. Suppose you were trying to prove a statement concerning how a particular social welfare function F would rank alternative *a*, relative to alternative *b*, in the social preference ordering corresponding to a particular profile P1. And suppose you knew the following: (1) another profile, P2, is just like P1 except that one of the individuals ranks alternative *c* lower in P2 than in P1 and (2) F specifies a social preference ordering for P2 that has *a* ranked above *b*. If you wanted to prove that F specifies a social preference ordering for P1 that has *a* ranked above *b*, is there a particular condition you could appeal to, in order to prove that? (That is, is there a particular condition such that, if F satisfies that condition, then the desired statement follows?) If so, which one?

questions due Monday, April 28 (section 17):

85. Let C be the claim that if the number of alternatives is at least 3 and the number of people is at least 2, no social welfare function satisfies conditions U, I, M, NI, ND, and P. Does C imply Arrow's impossibility theorem, or is C implied by Arrow's impossibility theorem, or neither?

86. On what basis can we claim that if a social welfare function satisfies conditions U, I, M, and NI, then it satisfies conditions U, I, M, NI, and P?

87. Suppose set *S* is decisive for alternative *k* over alternative *r*. What do we need to know about some profile in order to legitimately infer that *k* is ranked above *r* in the social preference ordering corresponding to that profile?

questions due Wednesday, April 30 (section 18):

88. Consider the following profile for a two-person society:

<u>A</u>	<u>B</u>
c	*b*
a	*d*
b	*c*
d	*a*

Suppose conditions P and L are in force, and person 1 has control (in the condition-L sense) over *a* versus *b*, and person 2 has control (in the condition-L sense) over *c* versus *d*. Does this profile show that condition U cannot be satisfied? Why or why not?

89. Suppose person 1 has control (in the condition-L sense) over *e* versus *f* and person 2 has control (in the condition-L sense) over *g* versus *h*. What is a profile that would show that it is impossible for a social welfare function to satisfy conditions U, P, and L?

90. As in the previous question, suppose person 1 has control (in the condition-L sense) over *e* versus *f* and person 2 has control (in the condition-L sense) over *g* versus *h*. What is a profile that would *fail* to show that it is impossible for a social welfare function to satisfy conditions U, P, and L?