Ben Eggleston
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Philosophy 666: Rational Choice Theory
April 1, 2016

test 2 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Questions:

1. Let Lottery F comprise a 70-percent chance of winning \$100 and a 30-percent chance of winning \$0.
   Let Lottery G comprise a 50-percent chance of winning \$100 and a 50-percent chance of winning \$0.
   Which of the following is required by the reduction-of-compound lotteries condition?

   (a) Lottery F I \$70

   (b) Lottery F P Lottery G

   (c) Lottery F P L(1/2, Lottery F, Lottery G)

   (d) L(1/2, Lottery F, Lottery G) I L(60/100, \$100, \$0)

   answer: d

2. Suppose you are predicting the preferences of a person named Wayne, whose preferences are assumed to satisfy the rationality conditions, with the set of prizes being $\{a, b, c, d, e\}$. Suppose Wayne's favorite prize is a and his least-favorite prize is e, and after asking him questions that relate to the continuity condition, you find that his values of p for prizes b, c, and d, respectively, are 3/4, 2/5, and 1/3. If you had to predict Wayne's preference between two options, and one of them was the lottery L(1/2, b, c), what lottery involving no prizes other than a and e would the substitution condition entitle you to convert that into? (If it is a compound lottery, leave it like that; i.e., do not reduce it.)

   answer: L(1/2, L(3/4, a, e), L(2/5, a, e))

3. Suppose that a certain game is played by player 1 choosing $x = 1$ or $x = 2$, then player 2 (with knowledge of player 1's choice) choosing $y = 5$ or $y = 6$, then payoffs to both players being some function of x and y (which is common knowledge between the players). Give an example of one of player 1's strategies, and state how many strategies player 1 has. Then give an example of one of player 2's strategies, and state how many strategies player 2 has.

   answer:

   One of player 1's strategies is 'set $x = 1$', and player 1 has two strategies.

   One of player 2's strategies is 'If $x = 1$, then set $y = 5$, and if $x = 2$, set $y = 5$', and player 2 has four strategies.

4. Analyze the following game using dominance considerations and write the strategy pair(s) corresponding to its solution(s). Write each strategy pair in the form (R_x, C_y), where *x* and *y* are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3	C_4
R_1	4	5	4	6
R_2	9	5	6	2
R_3	8	7	6	7

answer: (R_3, C_3)

5. State whether the following game has any equilibrium strategy pair(s). (You can ignore mixed strategies and focus on pure strategies only.) If it does, write it (or each of them) in the form form (R_x, C_y), where *x* and *y* are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3	C_4
R_1	3	8	5	1
R_2	6	7	6	8
R_3	4	2	3	9

answer: (R_2, C_1) and (R_2, C_3)

6. Draw the following matrix on one of your answer sheets and fill in each of the four cells with an integer between 1 and 9 (you can use up to four different integers, or you can repeat some integers) so that R_1 dominates R_2 but the outcome at (R_2, C_2) is an equilibrium outcome. (It does not have to be the only equilibrium outcome in your matrix.) Circle all of the equilibrium outcomes in your matrix.

	C_1	C_2
R_1		
R_2		

answer: the following, or many other possibilities:

	C_1	C_2
R_1	2	(1)
R_2	1	(1)

7. Derive the values of *p* and *q* that make (*p* R_1, (1 – *p*) R_2; *q* C_1, (1 – *q*) C_2) an equilibrium strategy pair for the following game. To derive each value, start with either the equation $EU(R_1) = EU(R_2)$ or the equation $EU(C_1) = EU(C_2)$ – whichever is appropriate – and show your work. Conclude each derivation with an equation of the form '*p* = __' or '*q* = __'.

	C_1	C_2
R_1	5	8
R_2	6	2

answer:

To derive p, we proceed as follows:

$$EU(C_1) = EU(C_2)$$

$$(p)(5) + (1-p)(6) = (p)(8) + (1-p)(2)$$

$$5p + 6 - 6p = 8p + 2 - 2p$$

$$-p + 6 = 6p + 2$$

$$-7p = -4$$

$$p = \frac{4}{7}$$

correction added December 22, 2019: In the second line of the equations above, the utilities should be the negations of the utilities given in the matrix, since the equations above refer to the utilities of the column player. Thus, the equations should appear as follows:

$$EU(C_1) = EU(C_2)$$

$$(p)(-5) + (1-p)(-6) = (p)(-8) + (1-p)(-2)$$

$$-5p - 6 + 6p = -8p - 2 + 2p$$

$$p - 6 = -6p - 2$$

$$7p = 4$$

$$p = \frac{4}{7}$$

Analogously, to derive q, we proceed as follows:

$$EU(R_1) = EU(R_2)$$

$$(q)(5) + (1-q)(8) = (q)(6) + (1-q)(2)$$

$$5q + 8 - 8p = 6q + 2 - 2q$$

$$-3q + 8 = 4q + 2$$

$$-7q = -6$$

$$q = \frac{6}{7}$$

correction added December 22, 2019: In the third line of the equations above, the '8p' should be '8q'.

8. What values of p and q make (p R_1, $(1 - p)$ R_2; q C_1, $(1 - q)$ C_2) an equilibrium strategy pair for the following game? (You do not have to show your work. An answer of the form 'p = __, q = __' can earn full credit.)

	C_1	C_2
R_1	6	4
R_2	2	5

answer: $p = 3/5$, $q = 1/5$

Use the following game for questions 9 and 10.

	C_1	C_2
R_1	3	7
R_2	5	1

9. What are the expected utilities of the row player's strategies R_1 and R_2, on the assumption that the column player is playing the mixed strategy (3/4 C_1, 1/4 C_2)? (Do two separate computations and show your work.) Could that mixed strategy for the column player be one half of an equilibrium strategy pair? Why or why not?

answer:

$$EU(R_1) = \frac{3}{4} \times 3 + \frac{1}{4} \times 7 = \frac{9}{4} + \frac{7}{4} = \frac{16}{4} = 4$$

$$EU(R_2) = \frac{3}{4} \times 5 + \frac{1}{4} \times 1 = \frac{15}{4} + \frac{1}{4} = \frac{16}{4} = 4$$

Yes, that mixed strategy for the column player could be one half of an equilibrium strategy pair, because it makes the row player indifferent between his two pure strategies and, consequently, indifferent among all of his strategies (including his mixed strategies).

10. What are the expected utilities of the column player's strategies C_1 and C_2, on the assumption that the row player is playing the mixed strategy (3/4 R_1, 1/4 R_2)? (Do two separate computations and show your work.) Could that mixed strategy for the row player be one half of an equilibrium strategy pair? Why or why not?

answer:

$$EU(C_1) = \frac{3}{4} \times 3 + \frac{1}{4} \times 5 = \frac{9}{4} + \frac{5}{4} = \frac{14}{4} = \frac{7}{2}$$

$$EU(C_2) = \frac{3}{4} \times 7 + \frac{1}{4} \times 1 = \frac{21}{4} + \frac{1}{4} = \frac{22}{4} = \frac{11}{2}$$

No, that mixed strategy for the row player could not be one half of an equilibrium strategy pair, because it makes the column player prefer to play C_2, rather than being indifferent among her two pure strategies and, consequently, indifferent among all of her strategies (including her mixed strategies).

(See correction on next page.)

correction added December 22, 2019: In the equations above, the utilities should be the negations of the utilities given in the matrix, since the equations above refer to the utilities of the column player. Thus, the equations should appear as follows. Then, in the text following the equations, the reference to C_2 should be a reference to C_1.

$$EU(C_1) = \frac{3}{4} \times (-3) + \frac{1}{4} \times (-5) = -\frac{9}{4} - \frac{5}{4} = -\frac{14}{4} = -\frac{7}{2}$$

$$EU(C_2) = \frac{3}{4} \times (-7) + \frac{1}{4} \times (-1) = -\frac{21}{4} - \frac{1}{4} = -\frac{22}{4} = -\frac{11}{2}$$

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.