

test 2 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions.

Questions:

For questions 1–6, let L1 be a lottery that provides a 1/4 chance of winning \$1,000 and a 3/4 chance of winning \$0. Also, assume the following:

$$u(\$350) = u(\$0) + x$$

$$u(\$1,000) = u(\$350) + y$$

$$x > 0$$

$$y > 0$$

1. Suppose Paul prefers \$350 to L1. What constraint concerning x and y (in addition to the constraints just stated) implies utility assignments for the three dollar amounts (\$0, \$350, and \$1,000) that make the principle of maximizing expected utility agree with this preference?

answer:

$$EU(\$350) > EU(L1)$$

$$u(\$350) > \left(\frac{1}{4}\right)u(\$1,000) + \left(\frac{3}{4}\right)u(\$0)$$

$$u(\$0) + x > \left(\frac{1}{4}\right)[u(\$0) + x + y] + \left(\frac{3}{4}\right)u(\$0)$$

$$4(u(\$0) + x) > u(\$0) + x + y + 3u(\$0)$$

$$4u(\$0) + 4x > 4u(\$0) + x + y$$

$$4x > x + y$$

$$3x > y$$

2. What are some utility assignments for the three dollar amounts (\$0, \$350, and \$1,000) that make the principle of maximizing expected utility agree with the preference attributed to Paul in question 1? Write your answer as a series of three equations, like this:

$$u(\$0) = _$$

$$u(\$350) = _$$

$$u(\$1,000) = _$$

(Of course, in each equation, instead of a blank ('_'), you will have a number.)

answer: the following, or many other possibilities:

$$u(\$0) = 0$$

$$u(\$350) = 1$$

$$u(\$1,000) = 2$$

3. Think again about Paul's preference for \$350 to L1. Does this indicate that he is risk averse, risk neutral, or risk seeking, or does it not indicate any of these? Explain your answer.

answer:

When a person prefers an amount of money to a lottery, we cannot infer that he is risk neutral or risk seeking; and we can infer that he is risk averse if and only if the lottery's expected monetary value is at least as great as the amount of money. (Otherwise, we can just explain the preference for the money as being based on a simple comparison between the amount of money and the expected monetary value of the lottery.) But in this case, the lottery's expected monetary value is less than the amount of money (so, we have no need to attribute risk aversion to him in order to explain his preference for the money). So, Paul's preference for \$350 to L does not indicate any of the three possibilities.

4. Let L2 be a lottery that is just like L1 except that the probabilities of 1/4 and 3/4 are replaced with probabilities of 1/5 and 4/5, respectively. And suppose Paul prefers L2 to \$350. What constraint concerning x and y (in addition to the constraints stated before question 1) implies utility assignments for the three dollar amounts (\$0, \$350, and \$1,000) that make the principle of maximizing expected utility agree with this preference?

answer:

$$EU(L2) > EU(\$350)$$

$$\left(\frac{1}{5}\right)u(\$1,000) + \left(\frac{4}{5}\right)u(\$0) > u(\$350)$$

$$\left(\frac{1}{5}\right)[u(\$0) + x + y] + \left(\frac{4}{5}\right)u(\$0) > u(\$0) + x$$

$$u(\$0) + x + y + 4u(\$0) > 5(u(\$0) + x)$$

$$5u(\$0) + x + y > 5u(\$0) + 5x$$

$$x + y > 5x$$

$$y > 4x$$

5. Consider the constraints concerning x and y that you derived as your answers to questions 1 and 4. Are there any values of x and y that satisfy both of them? Given that fact (whichever it may be), does this make the two preferences that the constraints are derived from similar to the preferences that the Allais paradox is concerned with, or different? (It is possible to get full credit for this question even if you answered question 1 and/or question 4 incorrectly, as long as you comment correctly on the constraints you derived. But if you are concerned that the constraints you derived will inhibit your answering this question correctly, you may make up some other constraints and comment on them as if they were your answers to questions 1 and 4.)

answer:

The two constraints are $3x > y$ and $y > 4x$. The first of these can be rewritten as $y < 3x$. Given that x is positive, $4x$ is greater than $3x$. Therefore, the two constraints jointly imply that y is less than some number ($3x$) but also greater than some larger number ($4x$). This is impossible. This makes the two preferences that the constraints are derived from similar to the preferences that the Allais paradox is concerned with, since those preferences also imply constraints that cannot both be satisfied.

6. Consider Paul's preference for L2 over \$350, and his preference for \$350 over L1. Also, assume Paul's preferences satisfy the transitivity condition. Therefore, we can infer that he prefers L2 to L1. Based on this fact and the way L1 and L2 are constructed, there is a particular rationality condition that we have strong reason to believe that Paul's preferences violate. Which condition is it? Explain your answer.

answer:

Paul's preference for L2 over L1 strongly suggests that he violates the better-odds condition. This is because L2 and L1 have the same prizes (\$1,000 and \$0) and the probability associated with the better prize is higher for L1 than for L2 ($1/4$ versus $1/5$). The better-odds condition would suggest that L1 should be preferred to L2. The fact that Paul prefers L2 to L1 strongly suggests he violates that condition.

7. Let lottery L3 comprise the following:
- a 70-percent chance at a lottery comprising a 60-percent chance of winning a mop and a 40-percent chance of winning a broom
 - a 30-percent chance at a lottery comprising an 80-percent chance of winning a mop and a 20-percent chance of winning a broom

What non-compound lottery does L3 reduce to? Show your work.

answer:

possible prizes: mop, broom

$$\text{mop probability} = \left(\frac{7}{10}\right)\left(\frac{6}{10}\right) + \left(\frac{3}{10}\right)\left(\frac{8}{10}\right) = \frac{42}{100} + \frac{24}{100} = \frac{66}{100}$$

$$\text{broom probability} = \left(\frac{7}{10}\right)\left(\frac{4}{10}\right) + \left(\frac{3}{10}\right)\left(\frac{2}{10}\right) = \frac{28}{100} + \frac{6}{100} = \frac{34}{100}$$

So, L3 reduces to the non-compound lottery comprising a 66-percent chance of winning a mop and a 34-percent chance of winning a broom, which can be written $L(66/100, \text{mop}, \text{broom})$.

8. Suppose Melinda is indifferent between a pen and $L(1/50, \text{typewriter}, \text{pencil})$. Which of the following is required of her by the substitution condition?
- (a) typewriter P pencil
 - (b) typewriter I pencil
 - (c) $L(1/2, \text{pen}, \text{paper}) \text{ I } L(1/2, L(1/50, \text{typewriter}, \text{pencil}), \text{paper})$
 - (d) $L(1/2, L(1/50, \text{typewriter}, \text{pencil}), \text{paper}) \text{ P } L(1/2, L(1/50, \text{typewriter}, \text{pencil}), \text{paper})$

answer: c

9. What is the role of the continuity condition in the proof of the representation theorem?
- (a) It facilitates the prediction of the chooser's preference regarding any two options by guaranteeing that the chooser has a most-preferred and a least-preferred prize.
 - (b) It facilitates the prediction of the chooser's preference regarding any two options by identifying, for any prize comprised by the options being compared, an equally desirable lottery involving only the chooser's most-preferred prize and least-preferred prize.
 - (c) It facilitates the prediction of the chooser's preference regarding any two options by guaranteeing that the chooser always prefers higher probabilities of getting their most-preferred prize.
 - (d) It facilitates the reconciliation of the chooser's predicted preference regarding any two options with the principle of maximizing expected utility by providing utilities for the two options (no matter how complex they may be) so that there is never any need to compute any options' *expected* utilities.

answer: b

10. What is something you were prepared to address or discuss on this test but were not asked about? Credit will be awarded in proportion to (1) the level of difficulty of what you write about, (2) how thoroughly you write about it, and (3) the absence of any statements in your answer that are incorrect or reflect any misconception about what you write about. In order keep the thoroughness criterion from impinging on the time you may need to answer the other questions on this test, any answer of at least 80 words will get full credit for thoroughness. (As a point of comparison, this question is 110 words long.)

answer: (many possible answers)

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.