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Philosophy 666: Rational Choice Theory
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name ________________________

test 3 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions.

Special instruction:

Below, matrices containing only one number in each cell represent zero-sum games in which the number given represents the row player's payoff and the column player's payoff is the negation of that.

Questions:

1. Suppose that a certain game is played by player 1 choosing x = 4 or x = 5, then player 2 (with knowledge of player 1's choice) choosing y = 8 or y = 9, then payoffs to both players being some function of x and y (which is common knowledge between the players). Give an example of one of player 1's strategies, and state how many strategies player 1 has. Then give an example of one of player 2's strategies, and state how many strategies player 2 has.

   answer:

   One of player 1's strategies is 'set x = 4', and player 1 has two strategies.

   One of player 2's strategies is 'If x = 4, then set y = 8, and if x = 5, set y = 8', and player 2 has four strategies.

2. Analyze the following game using dominance considerations and state the strategy pair that results from that analysis.

	C_1	C_2	C_3	C_4
R_1	3	3	5	4
R_2	5	8	1	4
R_3	5	7	6	6

   answer:

   First, eliminate R_1 and C_2:

	C_1	C_2	C_3	C_4
R_1	3	3	5	4
R_2	5	8	1	4
R_3	5	7	6	6

Second, eliminate R_2 and C_4:

	C_1	C_2	C_3	C_4
R_1	3	3	5	4
R_2	5	8	1	4
R_3	5	7	6	6

Third, eliminate C_3:

	C_1	C_2	C_3	C_4
R_1	3	3	5	4
R_2	5	8	1	4
R_3	5	7	6	6

The remaining strategy pair is (R_3, C_1).

3. State whether the following game has any equilibrium strategy pair(s). (You can ignore mixed strategies and focus on pure strategies only.) If it does, write it (or each of them) in the form (R_x, C_y), where x and y are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3
R_1	5	4	7
R_2	2	3	8

answer: (R_1, C_2)

4. Imagine a two-person zero-sum game in which both players have two pure strategies – R_1 and R_2 for the row player, and C_1 and C_2 for the column player. Suppose that when the row player plays a mixed strategy of the form (p R_1, $(1 - p)$ R_2) against a mixed strategy for the column player of the form (q C_1, $(1 - q)$ C_2), the row player's expected utility is $[p \times (14q - 5)] + (6q - 2)$. What mixed strategy should the column player play, if she would like to play a mixed strategy that could (along with a correctly chosen mixed strategy for the row player) be part of an equilibrium strategy pair? (Be sure to write a mixed strategy for the column player, not just the value of a variable.)

answer:

The column player should play the mixed strategy corresponding to the value of q that makes the row player's expected utility independent of the value of p. Given that the expression for the row player's expected utility has p multiplied by $14q - 5$, this latter expression should be set equal to 0. That yields the equation $14q - 5 = 0$, or $14q = 5$, or $q = 5/14$. So, the column player should play the mixed strategy ($5/14$ C_1, $9/14$ C_2).

5. Derive the values of p and q that make (p R_1, $(1 - p)$ R_2; q C_1, $(1 - q)$ C_2) an equilibrium strategy pair for the following game. To derive each value, start with either the equation $EU(R_1) = EU(R_2)$ or the equation $EU(C_1) = EU(C_2)$ – whichever is appropriate – and show your work. Conclude each derivation with an equation of the form 'p = __' or 'q = __'.

	C_1	C_2
R_1	9	4
R_2	3	5

answer:

To derive p, we proceed as follows:

$$EU(C_1) = EU(C_2)$$

$$(p)(-9) + (1-p)(-3) = (p)(-4) + (1-p)(-5)$$

$$-9p - 3 + 3p = -4p - 5 + 5p$$

$$-6p - 3 = p - 5$$

$$-7p = -2$$

$$p = \frac{-2}{-7}$$

$$p = \frac{2}{7}$$

Analogously, to derive q, we proceed as follows:

$$EU(R_1) = EU(R_2)$$

$$(q)(9) + (1-q)(4) = (q)(3) + (1-q)(5)$$

$$9q + 4 - 4q = 3q + 5 - 5q$$

$$5q + 4 = -2q + 5$$

$$7q = 1$$

$$q = \frac{1}{7}$$

6. What values of p and q make (p R_1, $(1 - p)$ R_2; q C_1, $(1 - q)$ C_2) an equilibrium strategy pair for the following game? (You do not have to show your work. An answer of the form 'p = __, q = __' can earn full credit.)

	C_1	C_2
R_1	5	7
R_2	6	2

answer:

$$p = \frac{2}{3}$$

$$q = \frac{5}{6}$$

(These can be computed as follows.)

$$p = \frac{6-2}{(6-2)+(7-5)} = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$$

$$q = \frac{7-2}{(7-2)+(6-5)} = \frac{5}{5+1} = \frac{5}{6}$$

7. In the following game, what is the expected utility, for the row player, of playing the strategy (p R_1, $(1 - p)$ R_2), on the assumption that the column player is playing the strategy (2/3 C_1, 1/3 C_2)? Your answer should be of the form $xp + y$, where x and y are real numbers.

	C_1	C_2
R_1	7	2
R_2	4	6

answer:

EU(p R_1, $(1 - p)$ R_2)

$= p \times EU(R_1) + (1 - p) \times EU(R_2)$

$= p \times [(2/3 \times 7) + (1/3 \times 2)] + (1 - p) \times [(2/3 \times 4) + (1/3 \times 6)]$

$= p \times (14/3 + 2/3) + (1 - p) \times (8/3 + 6/3)$

$= p \times 16/3 + (1 - p) \times 14/3$

$= p \times 16/3 + 14/3 + (-p \times 14/3)$

$= p \times 16/3 + 14/3 + p \times -14/3$

$= p \times 2/3 + 14/3$

$= (2/3)p + 14/3$

8. What must be true of x in order for the following game to be a coordination game?

	C_1	C_2
R_1	x, 2	6, 1
R_2	5, 0	7, 3

answer: $x > 7$

9. Write a 2 × 2 matrix that is an example of a prisoner's dilemma. Circle the equilibrium outcome(s).

answer: the following, or many other possibilities:

	C_1	C_2
R_1	3, 3	1, 4
R_2	4, 1	2, 2

10. Write the following matrix on one of your answer sheets and circle any equilibrium outcome(s). Also state whether this game is a coordination game, a prisoner's dilemma, or neither.

	C_1	C_2
R_1	4, 4	9, 5
R_2	5, 8	1, 2

answer:

	C_1	C_2
R_1	4, 4	9, 5
R_2	5, 8	1, 2

This game is a coordination game.

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.